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The progress of ideas

THE ACCURACY OF MACROECONOMIC FORECASTS BASED ON BAYESIAN VECTORIAL-AUTOREGRESSIVE MODELS. COMPARATIVE ANALYSIS ROMANIA-POLAND

Abstract

The aim of this research is to make predictions for macroeconomic variables like inflation rate, unemployment rate and exchange rate for Romania and Poland using BVAR models. The one-step-ahead forecasts cover the horizon 2011-2013. Direct forecasts were developed using three types of priors for data covering the period from 1990 to 2012: Minnesota priors, non-informative priors and natural-conjugate priors. The forecasts' accuracy assessment based on generalized forecast error of second moment put in evidence the superiority of Poland's predictions based on a BVAR(2) model, compared to Romania's ones based on a BVAR(4) model for differenced and stationary data series. For inflation rate the forecasts are rather inaccurate, but for Poland the Minnesota priors and for Romania the non-informative priors determined the most accurate predictions for unemployment rate and exchange rate on the horizon 2011-2012. It is very likely that these types of priors generate the best forecasts in 2013.

Key words: *BVAR models, forecasts accuracy, likelihood function, Minnesota prior, non-informative prior, Natural Conjugate prior*

1. Introduction

The Bayesian models of vector auto-regressive type (BVAR) were introduced later in literature as an alternative to vector auto-regressive models to solve disadvantages like: difficulties in choosing the most suitable variables, the best model or the suitable lag, forecasts with a very low performance.

The parameters of the BVAR models are seen as variables with a certain prior repartition. The coefficient of this prior distribution is known as hyper-parameter. For the prior value the vector of the parameter follows a multivariate normal repartition with two parameters: a known average and a known covariance matrix.

The objective of this research is given by the construction of forecasts based on BVAR models for some macro-economic variables (inflation rate, exchange rate, unemployment rate) for Romanian and Poland on the horizon 2011-2013. Moreover, the predictions' accuracy is assessed and we identified the best forecasts corresponding to certain utilized prior information.

2. BVAR models construction

First of all, we start from $the\ vector\ y_i$ with"m" variables. Each variable has "p" lags. The other variables (deterministic variables plus constant) are located in another vector denoted by y_{*i} with m* elements. The VAR model has the form given below:

$y_i = A(L)y_{i-1} + Cy*_i + e_i\ ,\ \ e_i \to (0, \sum_e)$
k= mq+m* (k- number of explanatory variables)
c=mk (c- number of parameters)

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The VAR model is showed in two forms that are equivalent (Im- identity matrix, α- mk vector, X- Tk matrix, Y and E- Tm matrices, y and e- mT vectors):

Y=XA+E (1)
y= (Im*X)α+e, $e \to (0, \sum_e * I_T)$ (2)
The likelihood function is: L(α, $\sum_e$)∞ $\frac{1}{\sqrt{|\sum_e * I_T|}}$ exp{$-0.5(y-(Im*X)\alpha)`(y-(Im*X)\alpha)$} (3)
A decomposition is realized: $\{(y-(Im*X)\alpha)`(y-(Im*X)\alpha)\}$=
$((\sum_e^{-/0.5} * I_T)y - (\sum_e^{-0.5} * X)\alpha_{ols})'((\sum_e^{-0.5} * I_T)y - (\sum_e^{-0.5} * X)\alpha_{ols}) + (\alpha_{ols}-\alpha)'(\sum_e^{-1} * X'X)(\alpha_{ols}-\alpha)$

In the end, the likelihood function is computed as the product of a Normal density for α and a Wishart density of $\sum_e^{-1}$.

L(α, $\sum_e$)∞ N($\alpha/\alpha_{ols,}$ $\sum_e$, X, y)* W($\sum_e^{-1}$/y, X, $\alpha_{ols,}$T-k-m-1)

Four variants of priors are described:
1. Non-informative priors for α and $\sum_e$
2. Normal prior for α with fixed $\sum_e$
3. Normal prior for α with non-informative prior for $\sum_e$
4. Conditionally conjugate prior.

The posterior density for α is Normal (the average is $\bar{\alpha}$ and the variance $\overline{\sum_\alpha}$=[$\overline{\sum_\alpha^{-1}} + (\sum_e^{-1} * X'X)^{-1}$). $\sum_\alpha$ could be randomly chosen or the OLS estimate could be chosen: $\sum_{e,ols} = \frac{1}{T-1} \sum_{t=1}^{T} e'_{t,ols}\, e_{t,ols}$ ($e_{t,ols} = y_i - (Im*X)\alpha_{ols}$.

$\bar{\alpha}=(X'X)^{-1}(X'Y)$= $[\sum_\alpha^{-1} + (\sum_e^{-1} * X'X)]^{-1}[\sum_\alpha^{-1}\bar{\alpha} + (\sum_e^{-1} * X))'y]$)

The posterior average for α uses the same form as an estimator based on Theil's mixed approach. The coefficients are restricted to stochastic form. Prior restrictions are assimilated to dummy values. The posterior estimator is based on sample prior information with the sample and the weights provided by precisions.

In literature there are 2 approaches for structurale form of VAR models. The Canova's one [Canova, 1995, p.15] was later developed by [Gordon și Leeper, 1994, p. 1228-1247], implying the utilization of Normal-Wishart distribution for the form with for reduced parameters. A conditional approach is used to extract the structural parameters using the identification restrictions. If Aj are the model parameters and $\sum_e = A_0^{-1}A_0^{-1\prime}$, then it results Aj=$A_0^{-1}\,Aj$. For an overidentified A0, the method neglects the restrictions.
Another structural form was proposed by [Sims și Zha, 1999, p.1113-1155], with non-singular A0 and $\bar{y}$ with determistic variables.
$A_0 y_t - A(l)y_{t-1} + C\bar{y}_t = e_t$, e_t-> (0,I)

The form of the posterior covariance matrix is: $\overline{\sum_\alpha}$=[$\sum_\alpha^{-1} + (X'X * \sum\ \ ^{-1})]^{-1}$

We should specify the prior variance and the prior mean. If all the parameters tend to zero, the prior mean should be null.

In Minnesota variant a prior for α is determined, the covariance matrix being replaced by an estimation:

$$\alpha \to N(\underline{\alpha}_{M_n}, \underline{V}_{M_n})$$

The prior mean or other elements in Minnesota case are fixed to null value in order to ensure the parameters that tend to zero and for avoiding the overestimation risk. If we work with growth rates, it is important to set $\underline{\alpha}_{M_n}$ to 0_{KM}. For level data, we use a prior average that reflects that the variables follow a random walk. The prior average is zero for all elements, excepting those that correspond to first lag of the dependent variable from each equation. These variables are fixed to 1.

The prior covariance matrix is diagonal. If $\underline{V_i}$ is the block from $\underline{V_{M_n}}$ corresponding to the K coefficients from equation i and $\underline{V_{i,jj}}$ are the diagonal elements, the Minnesota approach will be:

$\underline{V_{i,jj}} = \frac{\underline{a_1}}{r^2}$, for coefficients with own lag r, r=1,...,p

$\underline{V_{i,jj}} = \frac{\underline{a_2}}{r^2}\frac{\sigma_{ii}}{\sigma_{jj}}$, for coefficients with own lag r of the variable $j \neq i$, r=1,...,p

$\underline{V_{i,jj}} = \underline{a_3}\sigma_{ii}$, for explanatory variables' coefficients

The natural-conjugate priors suppose that the prior, the posterior and the likelihood function have the same family distributions. The form of natural-conjugate prior is:

$\alpha/\Sigma \rightarrow N(\underline{\alpha}\Sigma * \underline{V}\ )$ și $\Sigma^{-1} \rightarrow$W$(\underline{S^{-1}}, \underline{\vartheta})$, unde $\underline{S^{-1}}, \underline{\vartheta}, \underline{V}$ și $\underline{\alpha}$- the chosen hyper-parameters

The restricted VAR model is actually a linear regression, the covariance matrix of errors having a specific form. A general prior value without the restrictions of the natural-conjugate priors is the independent Normal-Wishart prior:

$p(\beta, \Sigma^{-1})$= p(β)p(Σ^{-1}), where $\beta \rightarrow N(\underline{\beta}, \underline{V_\beta})$ și $\Sigma^{-1} \rightarrow W(\underline{S^{-1}}, \underline{\vartheta})$.

We start from the standard form of a VAR model of order p:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \mu + \varepsilon_t$$

y_t- vector of endogenous variables (dimension: n*1)

μ- vector of constant terms (dimension: n*1)

ε_t- vector of errors (dimension: n*1)

The errors are independently, identically and normally distributed.

The matrices of coefficients have the dimension n*n.

For this type of model, [Ciccarelli și Rebucci, 2003, p. 78] identified the over-fitting problem (the number of parameters to estimate n*(np+1) increases geometrically by the number of variables while the increase is proportional with the number of lags). The Bayesian approach is more suitable because we do not know if some coefficients are null or not. For the vector of parameters we can associate probability distributions. The estimation supposes the knowledge of prior distribution and of the information in the data. Litterman (1986) made several observations regarding the use of BVAR models for macroeconomic data:

- Most of the macroeconomic time series include a trend;
- The recent lags have the major influence;
- The own lags of a variable influence it in a bigger proportion than the lags of other variables in the model.

  [Litterman, 1986, p. 25] started from a multivariate random-walk to define the prior distribution. Actually, the prior repartition is centered around the random walk ($y_{n,t} = \mu_n + y_{n,t-1} + \varepsilon_{n,t}$).

The following properties were identified by [Doan, 2007, p.16] for standard priors:

- The priors are flat (non-informative) for deterministic variables;
- The priors are independent and normally distributed for the lags of endogenous variables;
- The means of prior distributions are set to zero, excepting the first lag of the dependent variable in each equation (they are set to one).

Other priors have to be set for variances. The standard error of the estimate corresponding to variable *j* in equation *i* with the lag *l* is S(j,i,l): $S(j,i,l) = \frac{[\delta g(l) f(j,i)] s_i}{s_j}$.

- δ –hyper-parameter (overall tightness of the prior)

g(l)- tightness of lag 1 compared to lag l (g(l) decreases harmonically, $g(l) = l^d$). If the lag length increases, the tightness around the prior mean will increase.

f(j,i)- tightness of the prior on variable j compared to variable i in equation i.
f(j,i)= w_{ij}, for $i \neq j$
f(j,i)= 1, for i=j

A mixed estimation is used in this case. The information provided by the sample is combined with that of stochastic prior information. If we have a model based on N observations, including v priors, the mixed estimation is based on N+v observations. The v prior observations are weighted in accordance with the degree of tightness. If the priors are more diffuse, it is normally to assume that the BVAR estimators tend towards the ordinary least squares estimators.

If we start from a single equation of a VAR model ($y_t = X\alpha_t + \varepsilon_t$, var($\varepsilon_t$)=$\sigma^2 I$), the stochastic prior for it is: r=Rα+u. For a single equation, the estimate is:
$\hat{\alpha}$=(X'X+R'R)$^{-1}$(X'y1+R'r)

3. Forecasts based on BVAR models for macroeconomic indicators from Romania and Poland

For making the Bayesian estimation, the direct predictions based on BVAR(4) models with intercept for Romania take into account the previously estimated VAR models, the criteria of lag selection suggesting the values 4 and 2. We adapted the programme in Matlab provided by [Koop and Korobilis, 2010, p.26] for variables with stationarized data sets for inflation rate, unemployment rate and exchange rate for both countries. For Romania we used the RON/USD average exchange rate and for Poland zloty/EURO and a BVAR(2) model with intercept. The argument for choosing a certain lag is given by the construction of VAR models with the suitable lag. The data are provided by Eurostat, covering the period from 1990 to 2012.

Table 1

Direct point one-year-ahead predictions for Romania and Poland based on BVAR models with constant (horizon: 2011-2013)

Type of prior value	**Year**	**Romania**			**Poland**		
		Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)	Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)
Minnesota prior	2011	2.2404 (2.15)	1.1711 (0.584)	0.9807 (0.514)	1.003 (1.54)	1.227 (0.8569)	1.34 (0.7657)
	2012	3.8193 (1.279)	2.5614 (1.736)	2.5972 (3.458)	1.008 (1.482)	1.2768 (1.356)	1.4272 (0.673)
	2013	2.441 (0.494)	4.207 (3.85)	2.76 (3.57)	1.1 (0.879)	1.347 (2.02)	1.457 (0.78)
		Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)	Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)
Non-informative prior	2011	2.2551(2.31)	1.189 (0.631)	1.023(0.655)	1.103 (1.64)	1.235 (0.8759)	1.41 (0.784)
	2012	3.8976(1.932)	2.705 (1.882)	2.6652 (3.572)	1.112 (1.757)	1.2964 (1.649)	1.4367 (0.685)
	2013	2.554 (0.526)	4.33 (3.909)	2.89 (3.65)	1.23 (0.885)	1.356 (2.54)	1.4687 (0.839)
Natural-conjugate prior		Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)	Δ inflation rate(T+1)	Δ unemployment rate(T+1)	Δ exchange rate(T+1)

	2011	2.2633 (2.87)	1.189 (0.599)	0.9733 (0.537)	1.076 (1.76)	1.245 (0.8654)	1.365 (0.7738)
	2012	3.8566 (1.288)	2.5945 (1.749)	2.6743 (3.477)	1.012 (1.745)	1.2856 (1.465)	1.4376 (0.772)
	2013	2.538 (0.4589)	4.386 (3.874)	2.879 (3.784)	1.1675 (0.7658)	1.356 (2.32)	1.466 (0.855)

Source: Authors' calculations

As we can observe from the previous table, there are not significant differences between estimations based on the three types of priors. The standard errors are quite large, which suggests a rather high degree of uncertainty for these predictions. The predicted differences for inflation rate in Poland are lower than those for Romania.

In order to make a comparative analysis pf the forecasts' accuracy, we used as a measure of forecasts' accuracy the generalized forecast error of second moment (GFESM) [Bratu, 2012, p. 33]. This measure is computed as determinant of the expected value that corresponds to vector of predictions' errors for the future moments in the analysed horizon.

$$GFESM = \left| E\left| \begin{pmatrix} e_{t+1} \\ e_{t+2} \\ \cdots \\ e_{t+h} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} e_{t+1} \\ e_{t+2} \\ \cdots \\ e_{t+h} \end{pmatrix}^T \right| \right|$$

e_{t+h} - the n-dimensional forecast error corresponding to the model with n variables on the horizon h

Table 2

Values for generalized forecast error of second moment for predictions made for Romania and Poland (horizon 2011-2012)

Minnesota prior	Inflation rate	Unemployment rate	Exchange rate
Romania	39.74770449	5.65990996	4.82039872
Poland	11.579609	***0.778529***	***1.82988117***
Non-informative prior			
Romania	40.74970276	***1.356017***	***0.90785032***
Poland	***10.985809***	0.900225	2.53292717
Natural-conjugate prior			
Romania	***37.95538756***	8.1808	9.82280612
Poland	11.781044	1.560976	3.46691997

Source: Authors' calculations

According to the values of GFESM indicator, the BVAR model proposed for Poland generated for Poland on the horizon 2011-2012 more accurate predictions compared to Romania. The highest accuracy was registered for unemployment rate forecasts, the value of indicator being less than 1 in the case of natural-conjugate prior. The predictions for the inflation rate have the highest degree of uncertainty for both countries. In the category of indicators that were predicted for Romania, the highest accuracy was observed for the exchange rate anticipations. A very low accuracy was registered for predictions on the horizon 2011-2012 based on natural-conjugate prior. For Romania, under the assumption of non-informative priors, the most accurate predictions were obtained for unemployment rate and exchange rate. For the inflation rate in Romania, the natural-conjugate prior generated superior predictions compared to the other priors. In the case of Poland, Minnesota priors are the best for unemployment and exchange rate predictions and the utilisation of non-informative priors ensure the prognosis superiority for inflation rate on the horizon 2011-2012.

5. Conclusions

The BVAR models were developed in order to solve many problems generated by the use of VAR models in forecasting, one of them being the low accuracy problem. For the BVAR models that were proposed for Romania and Poland we obtained a superiority of predictions for Poland on the horizon 2011-2012. A possible explanation could be given by the fact that the predictions from a year to another for this country do not differ too much and there are not too large differences between the registered values.

We used 3 types of priors known in literature as Minnesota prior, non-informative prior and natural-conjugate prior and we identified that for unemployment rate and exchange rate forecasts in Poland, the Minnesota variant was the best, while for Romania the non-informative prior was the most suitable in forecasting.

The research limit might be done by the fact that it is likely that on the future horizon the hierarch of priors might change because of the real evolution of the macro-economic indicators, but on short horizon, for the next year, it is possible to have the same conclusions. The objectives of a future research could be related to the utilization of other priors or other estimation algorithms for BVAR models. We should mention that the built predictions are direct, but there is the possibility of constructing recursive forecasts.

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