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EPISTEMIC ANALYSIS OF METHODS USING ELEMENTARY TRIANGULAR FUZZY NUMBERS WITH ASSOCIATED INDICATORS

Abstract

The current scientific research is characterized by an advanced specialization, emphasis on increasing fragmentation of research topics.

In these conditions, the most results have a low degree of comparability and complexity of the subject studied is drastically reduced.

In this paper we propose an epistemic approach of the main methodologies for the quantification using triangular fuzzy numbers, emphasizing the possibilities of knowing the limits and influences entailed by the use of current techniques and the theoretical contingencies.

By using indicators of centers of weight associated to fuzzy number, we try to bring into question, through a multidisciplinary approach, the new possibilities of develop and complete the portfolio of instruments which can be used for decision-making.
Keywords: triangular fuzzy numbers, indicators of centers of weight associated to fuzzy number, decision-making

1. Introduction

The decision-making process under uncertainty conditions involves a referential system serving to evaluate, more or less objectively, the various types of actions which meet, to various degrees, the needs of the organisation, actions of which the system is rather indistinctly or vaguely aware. Most times, the actions experienced as being satisfactory will be accepted, reiterated and will become established, while actions experienced as unsatisfactory will be discarded. Over time, management will constantly exploit the solution or option which allows the optimal adjustment to a particular environment, ensures the highest degree of differentiation, provides the best diagnosis, and leads to the best treatment, with the most effective, real-time impact.

However, certain modern theoretical developments in management argue that the socio-human systems, under uncertainty conditions, resort to a simplified decision-making strategy, adopting the first satisfactory solutions that they are able to formulate coherently and accept by relative consensus. Gradually, specific criteria and methods have emerged, which provide a logical support for this type of decision-making. A key step is the steady development of distinct theories regarding fuzzy numbers, fuzzy sets, and fuzzy logic, applied to various fields based on the principles of interdisciplinary approach.

Fuzzy numbers open up new operational horizons, related to the mathematical models based on crisp numbers, and boast an increasingly broad applicability. Their broad applicability recommends them for interdisciplinary use in assessing various processes in the knowledge-based society in which uncertainty is a “constant” of the decision-making process.

The direct link with the uncertainty phenomenon is achieved through the concept of “approximate reasoning”, i.e. nuanced reasoning, which essentially underpins the logic of fuzzy numbers. It is viewed as the most general theory of incompleteness formulated so far,

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providing the opportunity for representation and reasoning with common formalisations, defined in ordinary manner, which explain its broad applicability in numerous fields.

In our undertaking, we will present the triangular fuzzy numbers, their associated indicators and the particular forms they can take. We thus provide a complex image of the various epistemic points of view used in theoretical developments based on elementary triangular fuzzy numbers, each of which present specific advantages and disadvantages.

2. Elementary triangular fuzzy numbers

Definition: A ***triangular fuzzy number*** expressed as $A=(a_L, a_M, a_R) \in F_{tr}$ is defined by its membership function $\mu_A: R \to [0,1]$, with the following form:

$$\mu_A = \begin{cases} \dfrac{x-a_L}{a_M-a_L}, daca & a_L < x < a_M \\ 1, \quad , daca & x = a_M \\ \dfrac{a_R-x}{a_R-a_M}, daca & a_M < x < a_R \\ 0, \quad daca & x \notin (a_L, a_R) \end{cases}, \text{ where } -\infty < a_L < a_M < a_R < +\infty$$

Graphically, figure 1 represents a triangular fuzzy number.

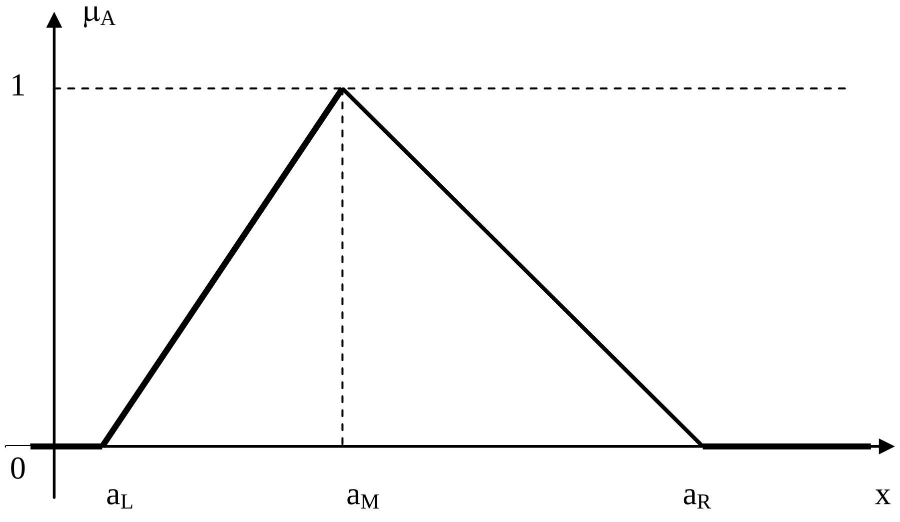

Fig. 1 Triangular fuzzy number

One may notice that μ_A is zero in the interval $(-\infty, a_L)$, increases linearly up to the value 1 on the segment $[a_L, a_M]$, decreases to the value 0 on the segment $[a_M, a_L]$, becomes zero for values in the range $(a_R, +\infty)$.

3. Classical operations with elementary triangular fuzzy numbers

Classic representation of triangular fuzzy numbers has next form :$T_a=(a_L,a_M, a_R)$.

For certain classes of problems is used the *L R* representation.

In order to enable basic operations with triangular fuzzy numbers ($T_a = (a_L, a_M, a_R)$, $T_a \in F_T$) the left representations, L(a), and right representations, R(a), of fuzzy numbers have been introduced:

T_a= [a_m, L(a) ; R(a)] where:

$$L(a) = a_M - a_L)$$
$$R(a) = a_R - a_M$$

In the article, we use only the classic form of representation.

Also there are used a few general notations:

- support: $S_p(T_a) = [a_L; a_R]$
- support leght: $L_{sp}(T_a) = a_R - a_L > 0$
- middle of support : $a_{sp}(T_a) = \frac{a_L + a_R}{2}$

Starting from the definitions put forward by Zadeh and based on generalisation, as proposed, for instance, by Kaufmann and Gupta, one may define the following mathematical operations for elementary triangular fuzzy numbers[2]: addition, subtraction, multiplication and division, illustrated in brief in next presentation.

Consider the next triangular fuzzy numbers: $T_c, T_a, T_b \in F_T$,

where : $T_c = [c_L; c_M; c_R]$; $T_a = [a_L; a_M; a_R]$; $T_b = [b_L; b_M; b_R]$ and $T_c = T_a \otimes T_b$.	Numerical example : $T_a = [2;3;4]$; $T_b = [3;5;6]$ $T_c = [2;3;4] \otimes [3;5;6]$

Adition (+)

$T_c = T_a(+)T_b \;=\; \begin{cases} c_L = a_L + b_L \\ c_M = a_M + b_M \\ c_R = a_R + b_R \end{cases}$	$T_c = [2+3;3+5;4+6] = [5;8;10]$;

Subtraction (-)

$T_c = T_a(-)T_b \;=\; \begin{cases} c_L = a_L - b_R \\ c_M = a_M - b_M \\ c_R = a_R - b_L \end{cases}$	$T_c = [2-6;3-5;4-3] == [-4;-2;1]$;

[2] Hans Schjaer-Jacobsen , *Modeling Economic Uncertainty*, Fuzzy Economic Review Volum IX, Nr. 2, November 2004, pg 53

Multiplication (*)

$$T_c = T_a(*)T_b$$

where
$$c_1 = \min(a_L b_L; a_L b_R; a_R b_L; a_R b_R)$$
$$c_M = a_M b_M$$
$$c_R = \max(a_L b_L; a_L b_R; a_R b_L; a_R b_R)$$

$$c_L = \min(2\cdot 3; 2\cdot 6; \cdot 4\cdot 3; 4\cdot 6)$$
$$c_M = 3*5$$
$$c_R = \max(2\cdot 3; 2\cdot 6; \cdot 4\cdot 3; 4\cdot 6)$$

$$T_c = [6;15;36];$$

Division (/)

$$T_c = T_a(/)T_b$$

where
$$c_1 = \min(a_L / b_L; a_L / b_R; a_R / b_L; a_R / b_R)$$
$$c_M = a_M / b_M$$
$$c_R = \max(a_L / b_L; a_L / b_R; a_R / b_L; a_R / b_R)$$

with condition
$$0 \notin [b_L; b_R]$$

$$c_L = \min(2/6; 2/3; 4/6; 4/3)$$
$$c_M = 3/5$$
$$c_R = \max(2/6; 2/3; 4/6; 4/3)$$

$$T_c = [2/6; 3/5; 4/3]$$

The listed definitions allow the demonstration of the existence of an approximate vector space in which not all traditional properties are observed. Even under such circumstances, this development based on fuzzy numbers is nowadays a widely used tool in decision-making processes under uncertainty conditions.

In our opinion, one of the major challenges lies in performing comparisons and ranking the solutions obtained. Most often, a hoped-for, expected optimal solution takes centre stage. To define it, one needs a complex interdisciplinary approach, which is difficult to achieve and sometimes strictly related to the nature of the problem under consideration, at other times incorporating less objective elements, requiring an active input from specialists. The concept of distance between fuzzy numbers is used at this point and solutions are ranked based on a sequence of comparisons. As the basis of these calculations, one can resort to a fuzzy number considered to be as follows:

– an „supremum”($T_s = [s_L; s_M : s_R]$)
– an „infimum” ($T_i = [i_L; i_M; i_R]$)
– A n „ideal” point of view ($T_e = [e_L; e_M; e_R]$) , different from previews numbers .

Can be used different approach for “distance” between fuzzy numbers[3]:

– Hamming distance :

$$\delta(T_a;T_b) = \int |\mu_a - \mu_b| dx$$

– Euclid distance :

[3] Arnold Kaufmann, Jaime Gil Aluja, *Tehnici speciale pentru gestiunea prin experţi,* Editura Expert , 1995

$$\varepsilon(T_a;T_b)=\int\sqrt{(\mu_a-\mu_b)^2}\ dx$$

– Minkowski distance

$$\rho(T_a;T_b)=\int\left(\sqrt{(\mu_a-\mu_b)^\lambda}\right)^{1/\lambda}\ dx$$

The (principal and secondary) ordering criteria are defined according to the ordering of the solutions, i.e. ascending or descending.

4. New trends in using operations with triangular fuzzy numbers

The need to simplify calculations and to increasingly facilitate technological transfer among various fields, within interdisciplinary approaches, has led to the development of solutions which rely on associated indicators.

A first step was the use of such associated indicators particularly in the process of comparing and ranking of solution. The outstanding benefits provided by this initial approach subsequently led to the use of indicators to define the multiplication and division operations.

In this paper, we will present a point of view, which, in our opinion, is easier to apply and more "palpable". It was developed by Gherasim who argued that the association of a indicator to an triangular fuzzy numbers to be used in all operations can simplify calculations. In the case of elementary triangular fuzzy numbers ($T_a=(a_L,a_M,a_R)$, $T_a\in F_T$) they take the following form[4]:

– Centre of weight (associated indicator):

$$G(T_a)=\langle T_a\rangle=\frac{a_L+2a_M+a_R}{4}$$

– Sign:

$$sign(T_a)=\begin{cases}sign(G(T_a)), daca & G(T_a)\neq 0\\ sign(a_M), daca & G(T_a)=0\end{cases}$$

The following basic arithmetical operations with triangular fuzzy numbers can be generalised: scalar multiplication, addition, subtraction, multiplication, division. In addition to the overall ordering indicator and the induced order are defined using associated indicators.

Consider the same triangular fuzzy numbers : $T_c,T_a,T_b\in F_T$,

$$G(T_c)=\langle T_c\rangle=\frac{c_L+2c_M+c_R}{4},$$

$$a_{sp}(T_c)=\frac{c_L+c_R}{2},$$

$$L_{sp}(T_c)=c_R-c_L>0$$

$$sign(T_c)=\begin{cases}sign(G(T_c)), daca & G(T_c)\neq 0\\ sign(c_M), daca & G(T_c)=0\end{cases}$$

$$\langle T_a\rangle=3$$

$$\langle T_b\rangle=4{,}75$$

$$a_{sp}(T_a)=3; \quad a_{sp}(T_b)=4{,}5 \quad \text{and}$$

$$L_{sp}(T_a)=2;\ sign(T_a)=+$$

$$L_{sp}(T_b)=3\ \ sign(T_a)=+$$

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[4] Ovidiu Gherasim, *Matematica incertitudinii* [EN: Mathematics of uncertainty], Editura Performantica, Iași 2004, p. 178

Multiplication by a scalar

$$k \cdot T_c = \begin{cases} (kc_L, kc_M, kc_R), & k > 0 \\ (kc_R, kc_M, kc_L), & k < 0 \end{cases}$$

$$3 \cdot T_a = (6;9;12);$$
$$(-2) \cdot T_b = (-12;-10;-6)$$

Adition (+)

$$T_c = T_a(+)T_b = \begin{cases} c_L = a_L + b_L \\ c_M = a_M + b_M \\ c_R = a_R + b_R \end{cases}$$

$$T_c = [2+3;3+5;4+6] = [5;8;10];$$

Subtraction (-)

$$T_c = T_a(-)T_b = \begin{cases} c_L = a_L - b_R \\ c_M = a_M - b_M \\ c_R = a_R - b_L \end{cases}$$

$$T_c = [2-6;3-5;4-3] == [-4;-2;1];$$

Multiplication (*)

$$T_c = T_a(*)T_b$$

where

$$T_c = \frac{T_a * \langle T_b \rangle + \langle T_a \rangle * T_b}{2}$$

$$T_c = [9.25;14.65;18.5];$$

Division (/)

$$T_c = T_a(/)T_b$$

where

$$T_c = \frac{T_a * \langle T_b \rangle + \langle T_a \rangle * T_b}{2\langle T_b \rangle^2}$$

with condition $0 \notin [b_L; b_R]$

$$T_c = [0.4;0.64;0.81]$$

Ranking criteria

Ranking triangular fuzzy numbers with associated weight centers shall be based on several successive criteria:

-*Weight ranking criteria:*

$$\begin{cases} G(T_a) > G(T_a) \Rightarrow T_a > T_b \\ G(T_a) < G(T_b) \Rightarrow T_a < T_b \end{cases}$$

-*Middle nucleus ranking criteria (if* ($G(T_a) = G(T_b)$:

$$\begin{cases} N(T_a) > N(T_b) \Rightarrow T_a > T_b \\ N(T_a) < N(T_b) \Rightarrow T_a < T_b \end{cases}$$

-Ranking criteria based on nucleus length and the sign:

$$if \left.\begin{matrix} G(T_a) = G(T_b) \\ N(T_a) = N(T_b) \end{matrix}\right\} \rightarrow \begin{cases} sign(T_a)L_{sp}(T_a) < sign(T_b)L_{sp}(T_b) \Rightarrow T_a > T_b \\ sign(T_a)L_{sp}(T_a) > sign(T_b)L_{sp}(T_b) \Rightarrow T_a < T_b \end{cases}$$

Conclusions

From a practical point of view, several reservations have been voiced about the use of fuzzy numbers. This is due both to difficulties in understanding fuzzy numbers and to the failure to promote these theories in the training programmes targeting specialists in various fields and top managers. This situation is compounded by the reluctant attitude of managers with significant practical experience in using fuzzy numbers and is also entrenched by cognitive inertia. The current crisis facilitates an overture to new methods of analysis and to interdisciplinary approaches.

In our view, fuzzy numbers allow the opening up of new perspectives in modelling decision-making situations characterised by irreducible or difficult to absorb uncertainties. Hence the increasingly active interest in the theoretical and practical aspects for these particular tools. In this paper, by presenting two ways of defining operations using fuzzy numbers in addition to a comparative analysis of the outcomes of their use, from a methodological perspective, we have demonstrated the ease of use of such theoretical constructs in modern management. We emphasise that the use of associated indicators in defining operations with fuzzy numbers simplifies such operations both in terms of facilitating understanding and of application. This addresses a growing trend observed among specialists in the field, namely the need to make the operations easier to implement and to expand the area of interdisciplinary applicability.

Modelling uncertainty by means of fuzzy numbers raises specific problems linked to their theory and application due to their complexity and difficulty. Despite these limitations, the method provides a new mode of scientific approach to uncertainty.

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