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METHODOLOGICAL ASPECTS REGARDING THE USE OF TRIANGULAR FUZZY NUMBERS WITH ASSOCIATED VARIABLE INDICATORS IN DECISION MAKING

Abstract: *The efficiency of the decision making process and the proper management of the situations of uncertainty require the intensification of the interdisciplinary theoretical approaches and the adaptation of methods from related fields. Uncertainty is no longer treated only as a factor that generates difficulties, but becomes an instrument in the management strategies, an opportunity for development.*

At the same time, the advances in the fuzzy theory allow the outline of new horizons in approaching the theoretical concepts in management, the development of existing methods and the outline of new lines of theoretical analysis. The use of fuzzy numbers with associated indicators is becoming increasingly important in developing decision making methods. In this context, the present article proposes the development of triangular fuzzy numbers with variable associated indicators and based upon specific elementary operations. Thus, a new theoretical direction for approaching the decision making processes and specific instruments is proposed, which is much closer to the practical mode, depending upon an index of uncertainty adsorption and management.

Key words: *fuzzy numbers, triangular fuzzy numbers with associated indicators, decision making.*

JEL: *D81, M21*

Introduction

Gradually, the uncertainty is no longer dealt with mainly through the concept of probability, a theoretical construct that has proven its advantages in analyzing past experiences and making forecasts, but also its limitations in capturing possible courses of action. More and more theoretical and applicative challenges are related to approaching decisions in conditions of uncertainty degree II or III, as these became increasingly important in the current economic and social context.

In order to absorb the persistent uncertainty (degree III), several management tools have been crystallized by which the entrepreneurs can manifest a logically structured attitude in extreme situations. In practice, one can resort to several categories of methods and techniques for managing this phenomenon: technocratic,

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political, structural or cultural, etc. Their effect upon the way the decision making process is structured is difficult to overcome by classical instruments, by crisp (real) numbers.

Therefore, the process of making decision under conditions of uncertainty required the development of new ways of its reduction or adsorption. A very important tool is the approach using fuzzy numbers.

In order to make operations with fuzzy numbers much easier, the use of associated indicators has been proposed: the center of gravity, the middle of the support and the core, the sign, the global indicator, etc.

To capture the potential of endogenous uncertainty, we propose (where the situation requires it, where the internal entropy of the system cannot be accurately or probabilistically determined) the use of fuzzy numbers with a variable center of gravity.

The variable character is captured by an associated coefficient - α which can take strictly positive and subunit values, between 0 and 1.

As its values experience a tendency toward 0, the fuzzy numbers capture a state of total ambiguity and pessimism internal to the system, which influences the evaluations made. On the contrary when the values of α experience a tendency towards 1, an optimistic state is expressed, where the mechanisms of absorption of uncertainty tend to cover a large part of the difficulties.

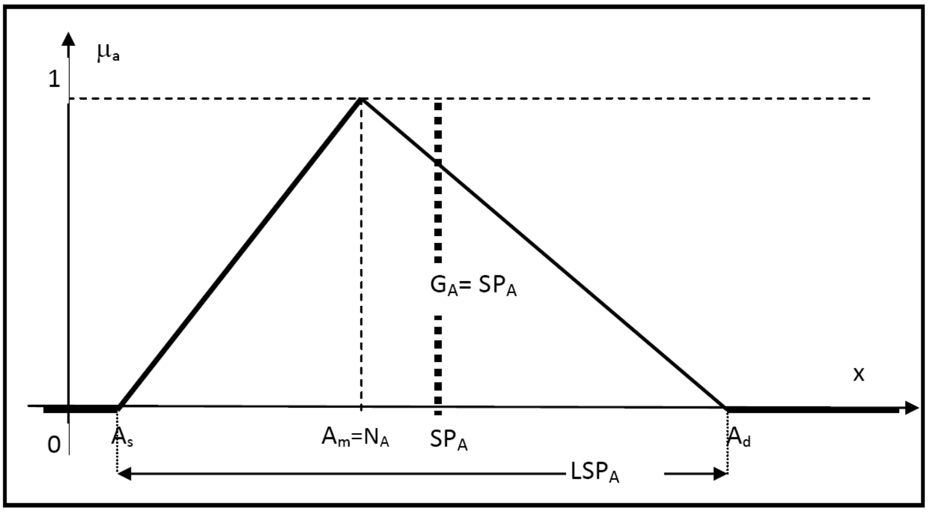
Triangular fuzzy numbers and associated indicators

A triangular fuzzy number (**NFT$_r$**) can be represented in different forms, starting from the definition given by Zadeh and up to the theoretical and practical developments utilized in the last decades, (of which Gherasim 2005, Tofan 2007, Maturo 2009, Jachobsen 2004, Wang 2009) are useful to our demonstration.

In view of the objective pursued, a triangular fuzzy number may be represented simply by an ordered triplet of the form $A = (A_s, A_m, A_d) \in$ **NFT$_r$** having the membership function $\boldsymbol{\mu_A: R \to [0,1]}$ defined as follows:

$$\boldsymbol{\mu_A(x) = \begin{cases} \frac{x-A_s}{A_m-A_s} & , A_s \le x \le A_m \\ 1 & , x = A_m \\ \frac{A_d-x}{A_d-A_m} & , A_m \le x \le A_d \\ 0 & , x \notin [A_s, A_d] \end{cases}} \tag{1}$$

Graphically, a triangular fuzzy number is represented according to Graph 1, where one can see the simple and synthetic associated indicators:



Graph 1. A triangular fuzzy number

In order to be able to work with these triangular fuzzy numbers, many simple or synthetic indicators have been defined. Among *the simple indicators* we recall:

The core (**which coincides with a_m**)	$N(A) = \{A_m\}$
The support	$Sp(A) = (A_s, A_d)$
The length of the support	$LSP_A = A_d - A_s \geq 0$
The middle of the core	$N_A = A_m$
The middle of the support	$SP_A = \frac{A_s+A_d}{2}$
Area to the left	$S_A^L = \int_{A_s}^{N_A} \mu_A(x)\mathrm{dx}$
Area to the right	$S_A^R = \int_{N_A}^{A_d} \mu_A(x)\mathrm{dx}$
Total area	$S_A = S_A^L + S_A^R$
The sign	$\delta_A = \begin{cases} sign(N_A) & , N_A \neq 0 \\ sign(A_m) & , A_m = 0 \end{cases}$

The synthetic indicators associated with a triangular fuzzy number

The synthetic indicators are real numbers associated with fuzzy numbers whose value belongs to the support, which are determined by the shape and size of the fuzzy numbers. The purpose of developing these sizes was to build a real synthetic image representative of a triangular fuzzy number, thus helping to ease the specific calculations.

Most times they have been developed in the form of central tendency indicators, or weight centers depending upon the simple indicators that define the triangular fuzzy number (for example, the triplet which defines a triangular fuzzy number). Among the most common and defining methods for synthetic indicators (G_A), there are as follows:

- ***the middle of the core*** $G_A = A_m$
- ***the middle of the support*** $G_A = SP_A = \frac{A_s + A_d}{2}$
- ***the expected value (average) of the nuance / estimate*** $G_A = \frac{A_s + A_m + A_d}{3}$
- ***the center of gravity (Gherasim 2004))*** $G_A = \frac{A_s + 2 * A_m + A_d}{2}$
- ***the center of gravity*** $G_A = N_A + \frac{S_{A-}^R S_A^L}{2}$
- ***the center of gravity (Tofan, 2011) (*** $\text{k} > 1)$ $G_A = \frac{k * A_s + 2(2-k)A_m + k * A_d}{2}$

From the viewpoint of the operationality of the theoretical and practical framework, a special approach is the development proposed by Gherasim (2004), where through the operations for the advanced fuzzy numbers the space is assumed and based upon 2 synthetic indicators: the center of gravity and the ordering indicator.

By analyzing how to define the center of gravity we note that starting from the general formula: $G_A = \frac{N_A + SP_A}{2}$ we arrive at a unitary and integrated approach for fuzzy rectangular, triangular, and trapezoidal numbers, where in particular we get the following representations:

- for rectangular fuzzy numbers (NFD), $(\forall)A = (A_m, A_M) \in \text{NFD}$

$$G_A = \ < A > \ = \frac{A_m + A_M}{2}$$

- for triangular fuzzy numbers (NFTr) $(\forall)\text{A} = (\text{A}_\text{s}, \text{A}_\text{m}, \text{A}_\text{d}) \in \text{NFTr}$

$$G_A = \ < A > \ = \frac{A_s + 2 \cdot A_m + A_d}{4}$$

- for trapezoidal fuzzy numbers (NFTp) $(\forall)\text{A} = (\text{A}_\text{s}, \text{A}_\text{m}, \text{A}_\text{M}, \text{A}_\text{d}) \in \text{NFTp}$

$$G_A = \ < A > \ = \frac{A_s + A_m + A_M + A_d}{4}$$

To allow the ordering of fuzzy numbers, in the same line, the global ordering indicator $O_A \in R$ associated with a triangular fuzzy number was constructed:

$$O_A = (G_A, N_A, \delta_A * LSP_A)$$

According to this indicator, two fuzzy numbers are ordered according to the size of the gravity centers, then by the middle of the cors, and then by the length of the support multiplied by the sign.

Center of variable gravity G_α associated to a triangular fuzzy number

The methods of management of the uncertainty of degree III have undergone constant developments through qualitative techniques of political, economic and social nature, aiming at: the artificial absorption of uncertainty; the artificial creation of uncertainty; the transfer of uncertainty; the obstruction or "strategic failure"; the reaching of the critical mass, etc.

The use of synthetic indicators associated with the fixed fuzzy numbers in the decision making processes related to the principle of mathematical expectation of the estimated values becomes insufficient to cover the broad spectrum of economic and social decision making situations.

The proposal is to develop a variable synthetic indicator that expresses the nature of the uncertainty absorption strategy estimated in the nuanced form (by the fuzzy number).

When discussing triangular fuzzy numbers, the variable associated indicator (see Alecu) was defined:

$$G_{A(\alpha)} = a_m + \alpha \cdot (a_M - a_m) = a_m \cdot (1-\alpha) + a_M \cdot \alpha \,,$$

Where: $\alpha \in [0;1]$ is on indicator of adosorption / management of uncertainty.

$G_{A(\alpha)}$ - represents the center of variable gravity associated with the rectangular fuzzy number.

A classic synthetic center of gravity formula is as defined below:

$$G_A = A_N + \frac{S_{A-}^R S_A^L}{2} \quad sau \quad G_A = A_N + \frac{1}{2}(S_A^R) - \frac{1}{2}(S_A^L)$$

By adding the coefficient of absorption of uncertainty $\alpha = \frac{1}{2} \in [0,1]$ one will easily obtain:

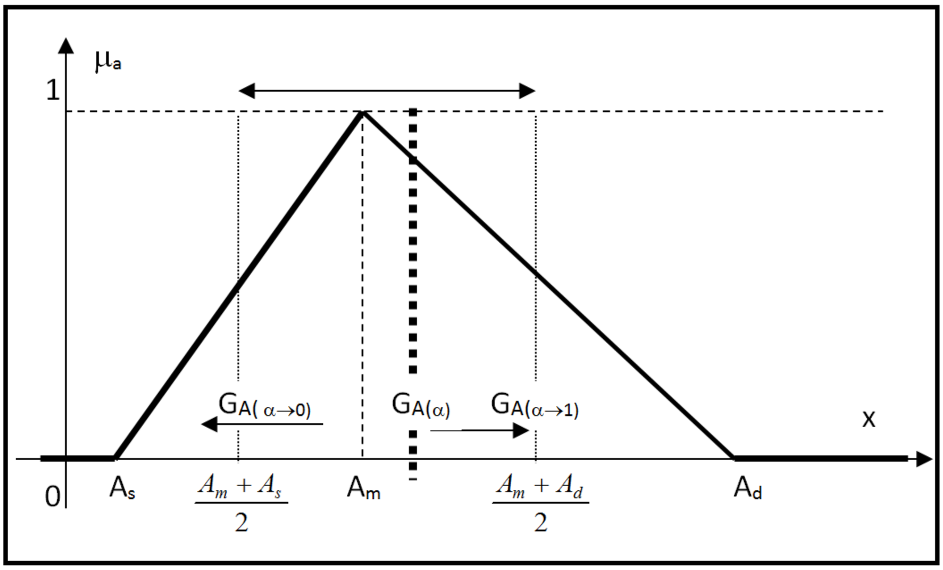
$$G_{A(\propto=\frac{1}{2})} = N_A + \propto (S_A^R) - (1-\propto)(S_A^L)$$

Starting from the general definition of the variable center of gravity associated with a general fuzzy number (Alecu, *Idei și Valori*, 2012), we propose as a synthetic way of defining the variable associated center of gravity for a triangular fuzzy number:

$$\boldsymbol{G_{A(\alpha)} = N_A + (\alpha - 1) \cdot S_A^L + \alpha \cdot S_A^R}$$

where: $\boldsymbol{\alpha \in [0, 1]}$ - is an indicator of adsorption / uncertainty management.

Graphically, a triangular fuzzy number with the associated variable center of gravity is represented in the following graph, where one can notice how the uncertainty and its management can be rendered by the associated indicator variability.



Graph 2. Triangular fuzzy number with variable associated center of gravity

By analyzing how the associated center of gravity can evolve one can identify three possible situations:

– a pessimistic uncertainty management attitude where the center of gravity $G_{\alpha\to 0}$ moves to the left on the segment $\left[\frac{A_s+A}{2}, A_m\right]$

– an optimistic uncertainty management attitude in which the center of gravity $G_{\alpha\to 1}$ moves to the right on the wing $\left[A_m, \frac{A_m+A_d}{2}\right]$.

– an equal attitude, which may be indifferent, an unresponsive attitude, or it can be a complex state of management focused upon the values most desired to be achieved. If one cannot undertake policies or strategies to absorb uncertainty, one is in a state of ambiguity in which the variable center will be the center of gravity.

By replacing simple indicators in the variable center of gravity formula for a triangular fuzzy number one will obtain:

$$\mathrm{G}_\alpha = \mathrm{A}_\mathrm{m} + (\alpha - 1) \cdot \frac{\mathrm{A}_\mathrm{m} - \mathrm{A}_\mathrm{s}}{2} + \alpha \cdot \frac{\mathrm{A}_\mathrm{d} - \mathrm{A}_\mathrm{m}}{2} \rightarrow$$

$$G_\alpha = \frac{A_m + (1 - \alpha) \cdot A_s + \alpha \cdot A_d}{2}$$

This can be transformed into a (very wide acceptance) useful form of praxis and uncertainty management methodology:

$$G_\alpha = \frac{(1-\alpha)\cdot(A_s + A_m) + \alpha\cdot(A_d + A_m)}{2}$$

Or, to put it more simply, in a specific acceptance of the complex conceptual framework of uncertainty management, applicable to any fuzzy number:

$$\boldsymbol{G_\alpha = (1-\alpha)\cdot(Nuancing\ to\ the\ left) + \alpha\cdot(Nuancing\ to\ the\ right)}$$

From the view point of the conceptual framework unit, we stress the fact that the customized use of the term of nuancing to the left / right instead of the term of variation to the left / right is not exclusive, but only equal, and it means the same thing. The concept of nuance integrates both the qualitative and quantitative variables. Or one can define the center of gravity for quantitative representations:

$$\boldsymbol{G_\alpha = (1-\alpha)\cdot(Variation\ to\ the\ left) + \alpha\cdot(Variation\ to\ the\ right)}$$

This simplified expression of the image of a fuzzy number will almost inevitably lead to new forms and developments of the synthetic indicators associated with the fuzzy numbers by which the management can carry out an approach to managing the absolute uncertainty as logically structured as possible.

If one analyzes the variation of the uncertainty adsorption coefficient, the situations identified for the center of variable weight are as follows:

a. For $\alpha \to 0$ (a pessimistic attitude) one will have:

$$G_{(\alpha\to 0)} \to [N_A - S_A^L] + 0\cdot S_A^R = A_N - S_A^L = A_m - \frac{A_m - A_s}{2} = \frac{A_s + A_m}{2}$$

b. For $\alpha \to 1$ (an optimistic attitude) one will have:

$$G_{(\alpha\to 1)} \to [N_A - 0\cdot S_A^L] + S_A^R = N_A + S_A^R = A_m + \frac{A_d - A_m}{2} = \frac{A_m + A_d}{2}$$

c. For a $\alpha \to \frac{1}{2}$ the center of variable gravity coincides with the (fixed) center of gravity:

$$\begin{aligned} G\_((\alpha \to 1/2)\,) &\to N\_A + (1/2-1)\cdot S\_A\hat{}L + 1/2\cdot S\_A\hat{}R \\ &= N\_A + 1/2\cdot S\_A\hat{}L + 1/2\cdot S\_A\hat{}R \\ &= N\_A + (S\_A\hat{}R - S\_A\hat{}L)/2 \end{aligned}$$

$$G_{\left(\alpha\to\frac{1}{2}\right)} \to \frac{A_s + 2\cdot A_m + A_d}{4}.$$

The ***sign of the variable center of gravity*** $\delta(G_\alpha$, associated with a triangular fuzzy number is defined as:

$$\delta(G_\alpha)=\begin{cases} sign\left((1-\alpha)\cdot\dfrac{A_s+A_m}{2}+\alpha\cdot\dfrac{A_d+A_m}{2}\right) & ,G_\alpha\neq 0\\ sign(A_m) & ,G_\alpha=0\end{cases}$$

Elementary operations with triangular fuzzy numbers using associated variable indicators

We will define the main operations such as adding, subtracting, multiplying and dividing by number intervals to which different variable centers of gravity have been associated.

A simplified way in defining a triangular fuzzy number with center of variable gravity A_α of is a fuzzy number $A_\alpha=(A_s, A_m, A_d)_\alpha\in NFT_r$ defined as a fuzzy set on R, having the membership function $\mu_A:R\rightarrow[0,1]$, in the following form:

$$\mu_a(x)=\begin{cases}\dfrac{x-a_s}{a_m-a_s} & ,a_s\leq x\leq a_m\\ 1 & ,x=a_m\\ \dfrac{a_d-x}{a_d-a_m} & ,a_m\leq x\leq a_d\\ 0 & ,x\notin[a_s,\ a_d]\end{cases}$$ where $A_s < A_m < A_d$,

to which an indicator of preference $\alpha\in[0.1]$ was associated as an expression of the way of absorbing the endogenous uncertainty manifested in the specific decision making process.

The ***center of variable gravity*** associated with a fuzzy number G is the real size $G_\alpha\in\mathbf{R}$, which is obtained by the following relation:

$$\boldsymbol{G_{A(\alpha)}=N_A+(\alpha-1)\cdot S_A^L+\alpha\cdot S_A^R}$$

The value of α does not influence the membership function of the fuzzy number, but only takes into account the affinity / inclination of the decision making subject to the uncertainty, the anxiety manifested within a specific decision making process assumed.

Let there be triangular fuzzy numbers with variable centers of gravity:

$A_\alpha=(A_s;A_m;A_d)_\alpha$, $B_\beta=(B_s;B_m;B_d)_\beta$ si $C_\gamma=(C_s;C_m;C_d)_\gamma$ ∈ NFT_r, where $\alpha,\beta,\gamma\in[0,1]$ the two assumed levels of absorption of uncertainty, the specific gravity centers with respectively $G_{A(\alpha)}$, $G_{B(\beta)}$ and $G_{C(\gamma)}$

We propose the following basic operations:

• ***Definition: The addition*** of two triangular fuzzy numbers $\mathrm{A}_\alpha, \mathrm{B}_\beta \in \mathrm{NFT}_\mathrm{r}$ with associated gravity centers according to $\alpha, \beta \in [0,1]$ is the law of composition $\oplus: \mathrm{NFT}_\mathrm{r} \times \mathrm{NFT}_\mathrm{r} \to \mathrm{NFT}_\mathrm{r}$, having the following form:

$$C_\gamma = \mathrm{A}_\alpha \oplus B_\beta \stackrel{\text{def}}{=} \begin{cases} (A_s + B_s; A_m + B_m; A_d + B_d)_\gamma \\ \gamma = (\alpha(A_s + B_s) + \beta(A_d + B_d))/(A_s + B_s + A_d + B_d) \end{cases}.$$

where: $\gamma \in [0,1]$ is the absorption coefficient of the uncertainty generated by addition.

One can notice that the assembly operation is a law of stable, associative and commutative composition.

• ***Definition: The multiplication of a triangular fuzzy number*** $\mathrm{A}_\alpha \in \mathrm{NFT}_\mathrm{r}$ with the associated center of gravity variable $\alpha \in NFT_r$ with a scalar $t \in \mathbf{R}$ is a triangular fuzzy number $C_\gamma = (C_s; C_m; C_d)_\gamma \in \mathrm{NFT}_\mathrm{r}$ of the form:

$$C_\gamma = t * \mathrm{A}_\alpha \stackrel{\text{def}}{=} \begin{cases} (t * A_s; t * A_m; t * A_d)_\gamma, & \gamma = \alpha, t \geq 0 \\ (t * A_s; t * A_m; t * A_d)_\gamma, & \gamma = 1 - \alpha, t < 0 \end{cases}$$

• ***Definition: The subtraction of two triangular fuzzy numbers*** $\mathrm{A}_\alpha, \mathrm{B}_\beta \in \mathrm{NFT}_\mathrm{r}$ with associated variable gravity centers according to $\alpha, \beta \in [0,1]$ is the law of composition $(-): \mathrm{NFT}_\mathrm{r} \times \mathrm{NFT}_\mathrm{r} \to \mathrm{NFT}_\mathrm{r}$, with the following form:

$$C_\gamma = \mathrm{A}_\alpha (-) B_\beta \stackrel{\text{def}}{=}$$

$$\begin{cases} \mathrm{A}_\alpha (-) B_\beta \stackrel{\text{def}}{=} (A_s - B_s; A_m - B_m; A_d - B_d)_\gamma \\ \gamma \stackrel{\text{def}}{=} (\alpha(A_s + B_s) + (1 - \beta)(A_d + B_d))/(A_s + B_s + A_d + B_d) \end{cases}.$$

We notice that the multiplication operation is a law of stable composition and the result was a triangular fuzzy number.

• ***Definition: The multiplication of two triangular fuzzy numbers*** $\mathrm{A}_\alpha, \mathrm{B}_\beta \in \mathrm{NFT}_\mathrm{r}$ with associated centers of gravity depending upon $\alpha, \beta \in [0,1]$ is the law of composition $\otimes: \mathrm{NFT}_\mathrm{r} \times \mathrm{NFT}_\mathrm{r} \to \mathrm{NFT}_\mathrm{r}$, having the following form:

$$C_\gamma = \mathrm{A}_\alpha \otimes B_\beta \stackrel{\text{def}}{=} \begin{cases} \dfrac{A_\alpha * G_{B(\beta)} + B_\beta * G_{A(\alpha)})}{2} \\ \gamma = (\alpha * \beta)/(\dfrac{\alpha + \beta}{2}) \end{cases},$$

$$= \begin{cases} (\dfrac{A_s * G_{B(\beta)} + B_s * G_{A(\alpha)})}{2}; \\ \dfrac{A_m * G_{B(\beta)} + B_m * G_{A(\alpha)})}{2}; \dfrac{A_d * G_{B(\beta)} + B_d * G_{A(\alpha)})}{2})_\gamma \\ \gamma = (\alpha * \beta)/(\dfrac{\alpha + \beta}{2}) \end{cases}$$

• ***Definition: The division of two triangular fuzzy numbers*** $A_\alpha, B_\beta \in NFT_r$ with associated variable centers of gravity according to $\alpha, \beta \in [0,1]$ is the law of composition $(/): NFT_r \times NFT_r \to NFT_r$ with the following form:

$$C_\gamma = A_\alpha(/)B_\beta \overset{\text{def}}{=} \begin{cases} \frac{A_\alpha * G_{B(\beta)} + B_\beta * G_{A(\alpha)})}{2*(G_{B(\beta)})^2} \\ \gamma = \frac{\alpha*(1-\beta}{\frac{\alpha+(1-\beta)}{2}}) \end{cases} ..$$

• Ordering ***criteria***

The ordering of the triangular fuzzy numbers with variable centers of gravity is done upon the basis of several successive criteria:

– the gravity center criterion $:\begin{cases} G_{A(\alpha)} > G_{B(\beta)} \to A_\alpha > B_\beta \\ G_{A(\alpha)} < G_{B(\beta)} \to A_\alpha < B_\beta \end{cases}$

If the centers of gravity do not achieve a clear separation of the two triangular fuzzy numbers, one should move on to the following criterion:

– ***the core means criterion*** $:\begin{cases} N_A > N_B \to A_\alpha > B_\beta \\ N_A < N_B \to A_\alpha < B_\beta \end{cases}$

If the centers of gravity and the means of the cores of two fuzzy triangular numbers are equal then the division will be made according to:

– the criterion of the sign lengths of the supports (cores):

$$\begin{cases} sign(\mathrm{A}) * LSP_A > \ sign(\mathrm{B}) * LSP_B \to A_\alpha > B_\beta \\ sign(\mathrm{A}) * LSP_A < \ sign(\mathrm{B}) * LSP_B \to A_\alpha > B_\beta \end{cases}$$

Conclusions

From the epistemological point of view we can say that the fuzzy numbers play a special role in the management of decisions through the contributions made in the study of the socio-economic phenomena, the laws governing thereof, the uncertainty and risks of their production, in analyzing the performances of the implemented strategies and programs, etc.

In the case of our paper, the focus was to develop a new model for making decisions in conditions of persistent uncertainty (degree III) starting from the formal or informal methods of managing uncertainty, by using triangular fuzzy numbers with associated variable indicators.

As one can notice, from a practical viewpoint, the triangular fuzzy numbers with associated variable indicators respond much better to the need for a logical argumentation for the selection of a course of action in conditions of a degree III of uncertainty; they allow the opening of new perspectives in modeling information on economic and social phenomena with great ease and they thus justify the growing interest for the theoretical and practical aspects of these specific instruments.

Among the main points of view underlying the importance of developing a variable gravity center associated to fuzzy numbers one can list: estimated information is utilized for uncertain statistically insufficient events (possibilities, not probable events); it captures certain environmental trends (growth, recession) etc.; it captures the potential informational entropies generated by internal evaluation errors of the system; it contributes to avoiding the excessive use of mathematical expectation; it captures the risk affinity or aversion of the person performing the assessment or appraisal; it captures the subjective mechanisms of absorption of uncertainty utilized in the management of the organization, and so on.

The variable gravity center associated of fuzzy numbers becomes an important construct for absorbing uncertainty in management techniques. Through a guided manipulation thereof, one can define a threshold in assuming one direction of action at the expense of another based upon long-term strategies a company is constantly investing in. This opens up new horizons for the theoretical approach of using these mathematical constructs in the management of uncertainty.

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