ELVIRA NAVAL[1] **Points of view**

ECONOMIC GROWTH MODELING UNDER GOVERNMENT POLICY UNCERTAINTY

Abstract: *In this paper a stochastic approach to the economic growth modeling, influenced by government expenditure, is proposed. Economic growth optimal control problem formulation in a stochastic form is concerned. Equilibrium growth rate was obtained using the stochastic maximum principle following new approach to the optimal control stochastic problem solution, in which the stochastic dynamic programming formulation takes the form of the maximum principle. This approach was applied to the solution of the stochastic optimal growth problem of how government policy, especially as far as corruption is concerned, influences economic growth.*

Key words: *Economic growth, Stochastic modeling, Stochastic maximum principle, Ito's processes, Ito's Lemma, Hamilton-Jacobi-Bellman equation, Government expenditure, Government policy, Corruption.*

Mathematics subject classification: *C61, D91.*

1. Introduction

Economic growth is the basic component of the sustainable social and economic development in any country, especially in developing countries like Republic of Moldova. Therefore, effective government polices in this direction are very imoportant. Optimal utilization of the government expenditure in order to minimize individual theft from it is a main problem to solve. It is well known that corruption has substantial, negative effects on economic growth. So, examination of the optimal economic growth problem in respect with government spending and its impact on the rate of economic growth is the main objective of this paper. In this direction some applications referred to the Republic of Moldova economic growth were examined in [1–2]. In the present paper the same problem of multiple equilibrium will be discussed, and for its solving the method [3] will be applied.

2. Problem formulation

As in [4–5] the model of multiple equilibrium in corruption and economic growth in stochastic formulation will be examined. Suppose that a N individuals denoted by $i \in (1, ..., N)$ are aimed to maximize an objective function

[1] Associate Professor, PhD, Institute of Mathematics and Computer Sciences, e-mail: elvira.naval@gmail.com

$$\mathbf{max}Q = E\left[\int_0^\infty e^{-\rho t}\left(\frac{c_{it}^{1-\sigma}}{1-\sigma}\right)dt\right] \tag{1}$$

supposed to following restrictions

$$\begin{gathered} dk_i = dt\left\{(1-\tau)\left[wl_i + rk_i\right] - c_i + S_i g\phi\left(\overline{S}\right)\right\} + u_i dz, \\ L_i + S_i = 1, \forall i \in (0,1,...,N) \end{gathered} \tag{2}$$

where E is an expectation operator, and c_i is particular consumption of the individual i, and σ is the value inverse to the intertemporal elasticity of substitution. It is assumed that population is constant over time and normalized to 1. At every moment of time one unit of labor services of each individual are allocated between productive work, L, and theft from the government, S: $L_i + S_i = 1$, $L_i \geq 0$, $S_i \geq 0 \; \forall \; i \in (1, \ldots, N)$. Government expenditure takes part from production function as in [6] and may be suited by rent seekers, who are able either to consume the proceeds or to invest them in their own firms. The total amount of resources that are extracted by individual i is $S_i G\phi(\overline{S})$, $\phi'(.) > 0$, $0 < \phi(\overline{S}) < 1 \forall \overline{S} \in (0,1)$, here $\overline{S} = \int_0^1 S_i di.$ So, this amount depends on the time that i spends stealing, S_i, and the amount of productive government expenditure available, G. Here $\phi(\overline{S})$ represents the proportion of stolen resources actually kept by the rent seeker. It is assumed that φ is a positive function in respect with $\overline{S}$, the total rent steeling activity in the economy. The production function for firm i is

$$\frac{dK_i}{dt} = (1-\tau)\left[wL_i + rK_i\right] - C_i + S_i G\phi\left(\overline{S}\right) \tag{3}$$

$$\begin{gathered} dK_i = dt\left\{(1-\tau)\left[wL_i + rK_i\right] - C_i + S_i G\phi\left(\overline{S}\right)\right\} = f(K_j, C_j)dt \\ dk_i = f(k_j, c_j)dt + u_i dz \end{gathered} \tag{4}$$

Here k_i, c_i are capital and consumption per capita respectively and dz is a stochastic Wiener process, u_i is a given vector function, and

$$f(k_j, c_j) = (1-\tau)\left[wL_i + rk_i\right] - c_i + S_i g\phi\left(\overline{S}\right).$$

Balanced wage w and rental rate r are determined through the marginal product of the labor and capital, respectively:

$$w = \frac{\partial Y_j}{\partial L_j} = \alpha K^{1-\alpha} L^{\alpha-1} G^\alpha \left[1 - \overline{S}\phi\left(\overline{S}\right)\right]^\alpha = \alpha\frac{Y}{L},$$

$$r = \frac{\partial Y_j}{\partial K_j} = (1-\alpha)K^{-\alpha}L^{\alpha}G^{\alpha}\left[1-\bar{S}\phi(\bar{S})\right]^{\alpha} = (1-\alpha)\frac{Y}{K}.$$

Where G is government expenditure, K_i is the capital stock belonging to firm i, $\bar{S}\phi(\bar{S})$ is the amount stolen that fails to reach the production processes as an input. It is assumed [5] that $\tau = G/Y =$ constant, where τ is the constant tax rate. In per capita indicators, production function looks as: $y_j = k_j^{1-\alpha} l_j^{\alpha}\left\{g\left[1-\bar{S}\phi(\bar{S})\right]\right\}^{\alpha}$.

So, capital per capita belonging to firm i in the stochastic case evolves according to

$$dk_i = dt\left\{(1-\tau)\left[wl_i + rk_i\right] - c_i + S_i g\phi(\bar{S})\right\} + u_i dz,$$

$$dk_i = f(k_j, c_j)dt + u_i dz,$$

where dz is a stochastic Wiener process.

3. Methodology and data sources

In this study economic growth methodology was applied. Government spending policy and state functionary corruption have been examined. Stochastic optimal control problem was formulated. Hamilton-Jacobi-Bellman principle, dynamic optimization methods have been used.

4. Problem solution

Now, the stochastic optimal control problem may be formulated as maximization of the objective function Eq. (1) subjected to the restriction following from Eq. (2): the respective optimality conditions now are:

$$0 = \max_{c_j, L_i, S_i}\left[e^{-\rho t}\left(\frac{c_{it}^{1-\sigma}}{1-\sigma}\right) + \frac{1}{dt}E(dL)\right] \tag{5}$$

and the corresponding HJB (Hamilton-Jacoby-Bellman) equation [8–9] becomes:

$$0 = \max_{c_j}\left[F + \frac{dL}{dt} + \frac{dL}{dk_j}f + \frac{1}{2}u^2\frac{\partial^2 L}{(\partial k_j)^2}\right] = \max_{c_j}\left[F + L_t + L_k f + \frac{1}{2}u^2 L_{k_j k_i}\right], \tag{6}$$

(4) are the corresponding optimality conditions.

The Hamiltonian function H, for the stochastic case is given as:

$$H = F + L_t + L_k f + \frac{1}{2} u^2 L_{k_j k_i} \quad f(k_i, c_i) = (1-\tau) w l_i + r k_i - c_i + Sg\phi(\overline{S})$$

second-order term appears from the consideration that the state variable vector $k = (k_1, k_2, \ldots, k_N)$, and the vector function u being given, are Itô's processes (Itô's Lemma) [6]. And the derivative of the (HJB) equation with respect to k,

$$L_{kt} + F_k + L_{kk} f + f_k L_k + \frac{1}{2} u^2 L_{kkk} + \frac{1}{2} \left(u^2\right)_k L_{kk} = 0, \tag{7}$$

therefore

$$L_{kt} + L_{kk} f + \frac{1}{2} u^2 L_{kkk} = -f_k - f_k L_k - \frac{1}{2} \left(u^2\right)_k L_{kk} \tag{8}$$

Also, using chain rule and considering second-order contribution of the derivatives with respect to k (Itô's Lemma), we obtain:

$$dL_k = \frac{\partial L_k}{\partial t} dt + \frac{\partial L_k}{\partial k} \frac{dk}{dt} dt + \frac{1}{2} \left(u^2\right)_k \frac{\partial^2 L_k}{(\partial k)^2} dk^2$$

Because from Itô's Lemma, $E\left[d(k^2) = u^2 dt\right]$ and substituting previous equation in the last one, we obtain:

$$\begin{aligned} \frac{dL_k}{dt} &= \frac{\partial L_k}{\partial t} + \frac{\partial L_k}{\partial k} \frac{dk}{dt} + \frac{1}{2} \frac{\partial^2 L_k}{(\partial k)^2} u^2 \\ \frac{dL_k}{dt} &= -F_k - f_k L_k - \frac{1}{2} \left(u^2\right)_k L_{kk} \end{aligned} \tag{9}$$

Deriving previous equation with respect to k, using chain rule and considering second-order contributions in the derivatives with respect to k (Itô's Lemma), we get:

$$dL_{kk} = \frac{\partial L_{kk}}{\partial t} dt + \frac{\partial L_{kk}}{\partial k} \frac{dk}{dt} dt + \frac{1}{2} \left(u^2\right)_k \frac{\partial^2 L_{kk}}{(\partial k)^2} dk^2$$

Then, Itô's Lemma, chain rule application and substituting Eq. (7) in Eq. (9), get:

$$\frac{dL_{kk}}{dt} = \frac{\partial L_{kk}}{\partial t} + \frac{\partial L_{kk}}{\partial k} \frac{dk}{dt} + \frac{1}{2} \left(u^2\right)_{kk} \frac{\partial^2 L_{kk}}{(\partial k)^2}$$

Equating adjoint variable μ to the first derivatives of the objective function L with respect to state variable k and ω as the second derivatives with respect to the state variable, the following is obtained:

$$\frac{d\mu}{dt} = -f_k\mu - \frac{1}{2}(u^2)_k\,\omega$$
$$\frac{d\omega}{dt} = -2\omega f_k - \mu f_{kk} - \frac{1}{2}(u^2)_{kk}\,\omega \tag{10}$$

Here $\mu = (\mu_1, \mu_2, ..., \mu_N)$ and $\omega = (\omega_1, \omega_2, ..., \omega_N)$ are two N-dimensional vectors.

Notice that the resulting problem is a 2n point boundary value problem. Summarizing for the stochastic case, the Hamiltonian function and the adjoint equations are:

$$H = \mu f + \frac{1}{2}u^2\omega$$

$$\frac{d\mu}{dt} = -f_k\mu - \frac{1}{2}(u^2)_k\,\omega,\quad \mu(T) = c \tag{11}$$

$$\frac{d\omega}{dt} = -2\omega f_k - \mu f_{kk} - \frac{1}{2}(u^2)_{kk}\,\omega,\quad \omega(T) = 0$$

5. Conclusions

In the present article the problem of correlations between slow economic growth and corruption for countries, especially those in transition, was formulated and considered. This problem is represented as stochastic optimal control one and its solution in stochastic formulation is obtained. In order to get the solution the derivation of the respective Hamilton-Jacobi-Bellman equation was applied. This method contributed to obtaining solution in the stochastic maximum principle form containing the first-order system of the conjugate differential equations. Note that the growth rate reached in stable condition for the examined problem is the same as for deterministic case, with one difference – there is an additional ordinary equation characterizing conjugate variable (shadow price of the stochastic restriction). In the future more complicated stochastic optimal control problem with the shock above all economy and with the shocks under productivity in intermediate good sectors will be studied.

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